Section 3-5

The Graph Scale-Change Theorem

Warm-up

1. Multiplying by three gives the same answer as dividing by $\frac{1}{3}$

2. Multiplying by .75 gives the same answer as dividing by $\frac{4}{3}$

3. Make a general statement that summarizes numbers 1 and 2 above.

Multiplying by n ($n \neq 0$) gives the same answer as dividing by $\frac{1}{n}$

Scale Change: A transformation that stretches or shrinks a graph both vertically and horizontally

Horizontal Scale Factor: The value that changes the horizontal values of a graph

Vertical Scale Factor: The value that changes the vertical values of a graph

Size Change: When the horizontal and vertical scale factors are the same

Graph Scale-Change Theorem

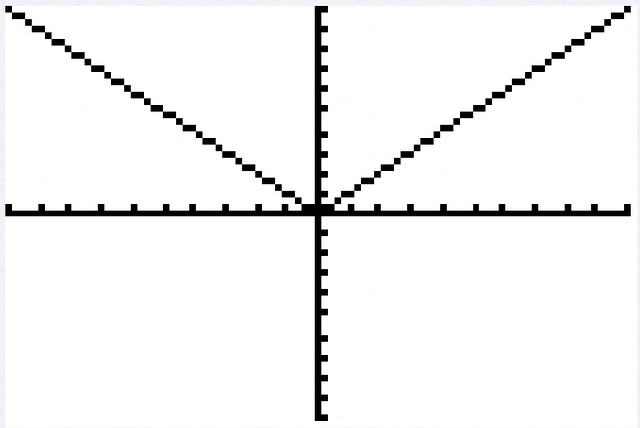
When dealing with a graph, the following transformations will give the same graphs:

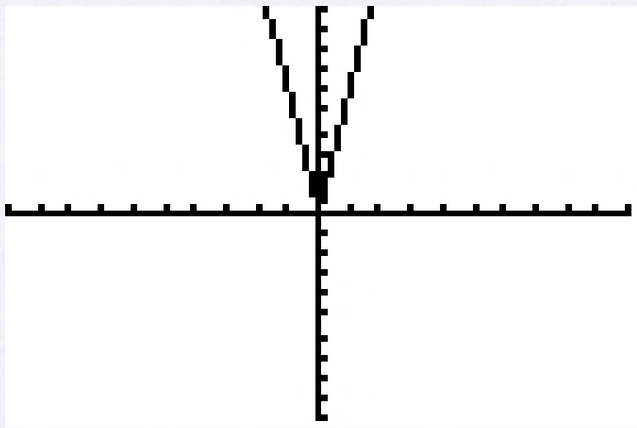
1. Replacing x with $\frac{x}{a}$ and y with $\frac{y}{b}$ in the equation

2. Applying the scale change $(x, y) \rightarrow (ax, by)$ to the graph, where a is the horizontal scale factor and b is the vertical scale factor

Example 1

Compare the graphs of $y = |x|$ and $y = |6x|$





X	Y1
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3

X= -3

X	Y1
-3	18
-2	12
-1	6
0	0
1	6
2	12
3	18

X= -3

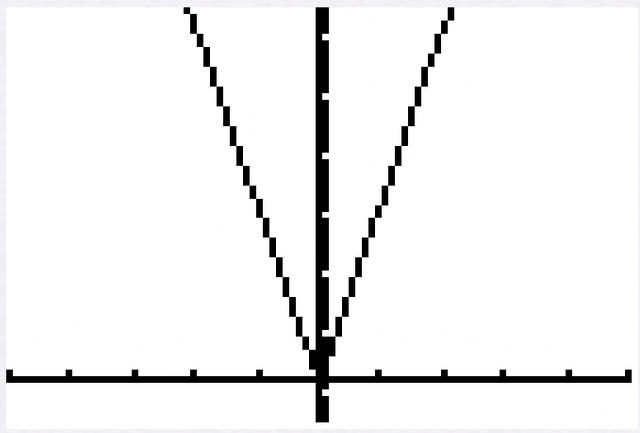
So, what can we say?

The graph of the second is the image of the graph of the first under a horizontal scale change of magnitude $\frac{1}{6}$.

This will shrink the graph horizontally. The angle at the vertex becomes acute, but the vertex does not change. The *y*-coordinates will now be 6 times as large for the corresponding *x*-coordinates.

Example 2

Sketch the graph of $\frac{y}{4} = |6x|$



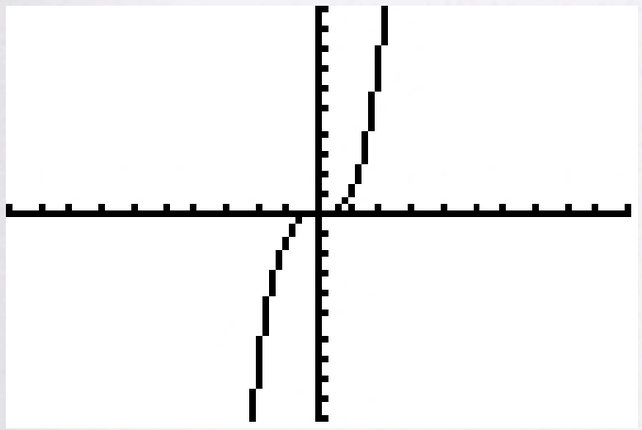
```
WINDOW
 Xmin=-5
 Xmax=5
 Xscl=1
 Ymin=-5
 Ymax=50
 Yscl=1
 Xres=1
```

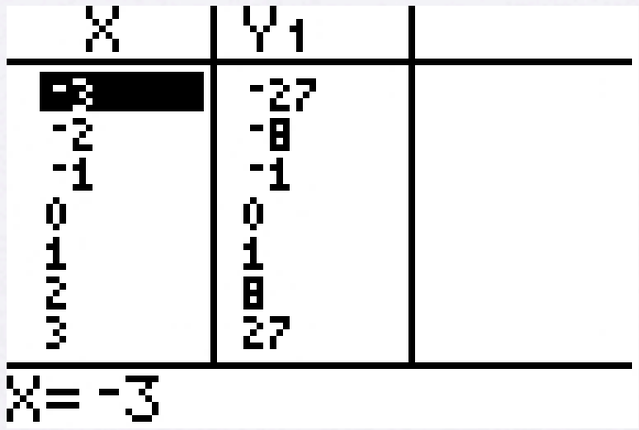
Negative Scale Factor:

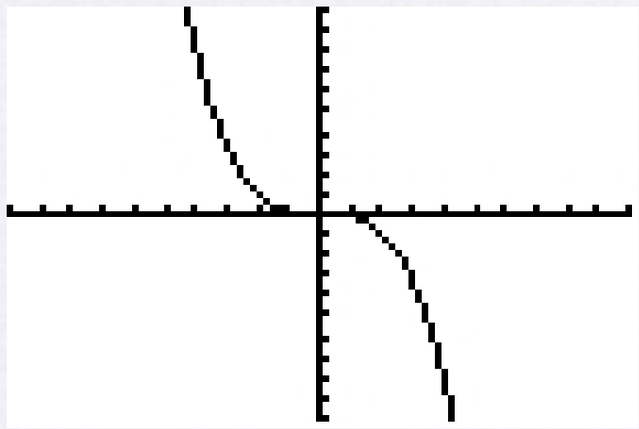
When using a negative scale factor, the values will be reflected over an axis

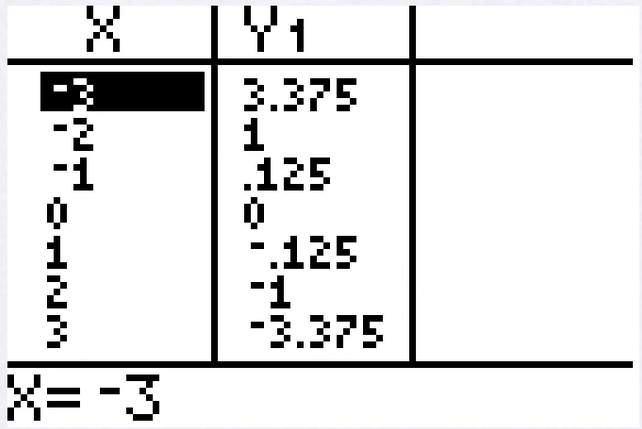
Example 3

a. Sketch the image of $y = x^3$ under $S(x, y) = (-2x, y)$









Example 3

b. Give an equation for the image

Since the *x*-coordinate is being multiplied by -2 in the scale change, we take the opposite for the equation. Thus, we need to divide *x* by -2.

$$y = \left(\frac{x}{-2}\right)^3$$

$$y = -\frac{1}{8}x^3$$

Example 4

The line $41x - 29y = 700$ contains the points (39, 31) and (10, -10). Use this information to obtain two points on the line with equation $20.5x - 87y = 700$.

x has changed by: 1/2

y has changed by: 3

$$S(x, y) = (2x, y/3)$$

$$(2\times39, 31\div3) = (78, 31/3)$$

$$(2\times10, -10\div3) = (20, -10/3)$$

Homework

p. 191 # 1 - 16, skip 13